Algorithms for NLP

Classification I

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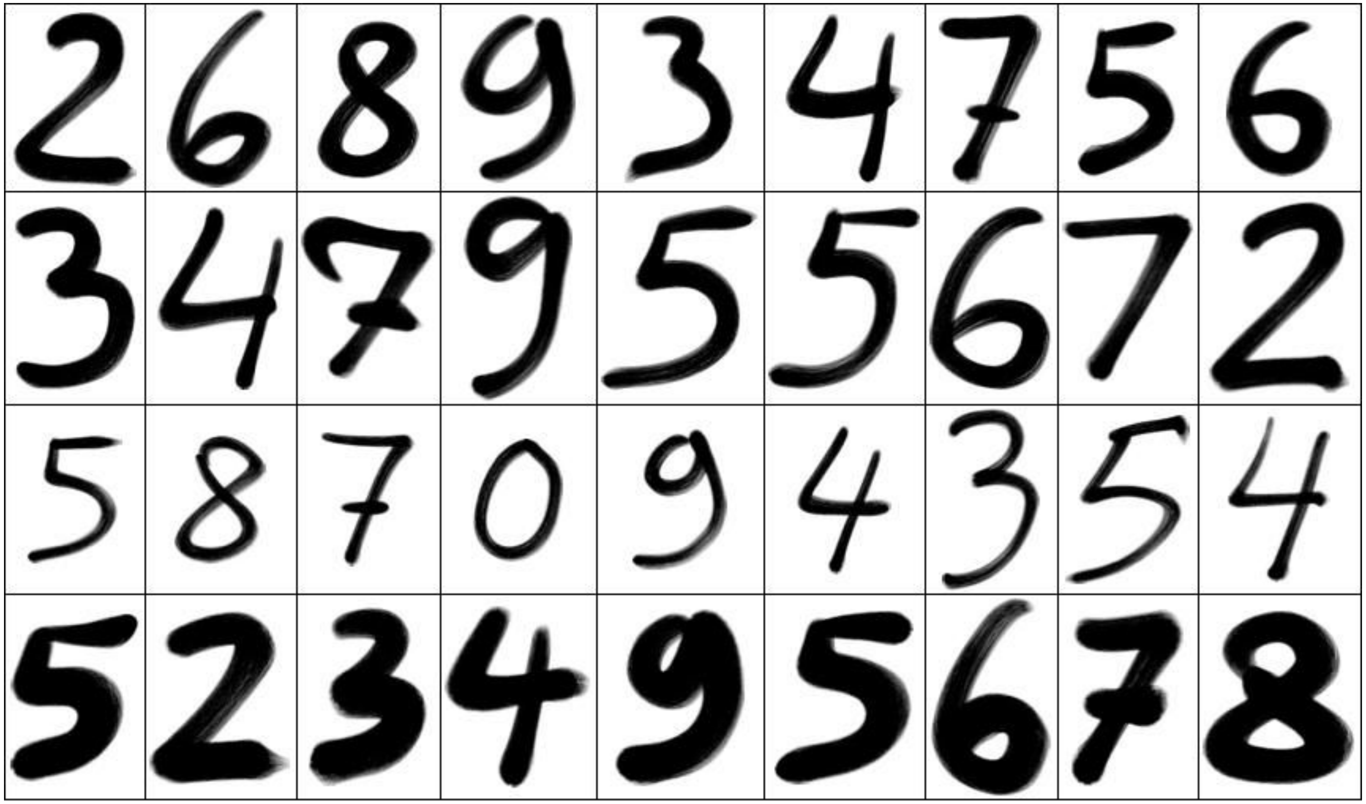
Classification

Image → Digit



Classification

Document → Category

Classification

Query + Web Pages → Best Match

“Apple Computers”

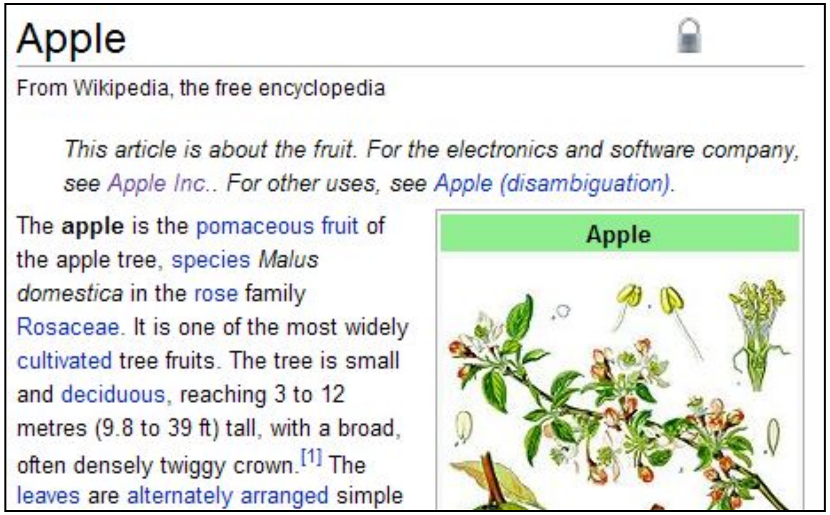
Apple Inc.

From Wikipedia, the free encyclopedia

(Redirected from Apple Computer)

Apple Inc.,

Apple Inc.



Apple

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This article is about the fruit. For the electronics and software company, see Apple Inc.. For other uses, see Apple (disambiguation).

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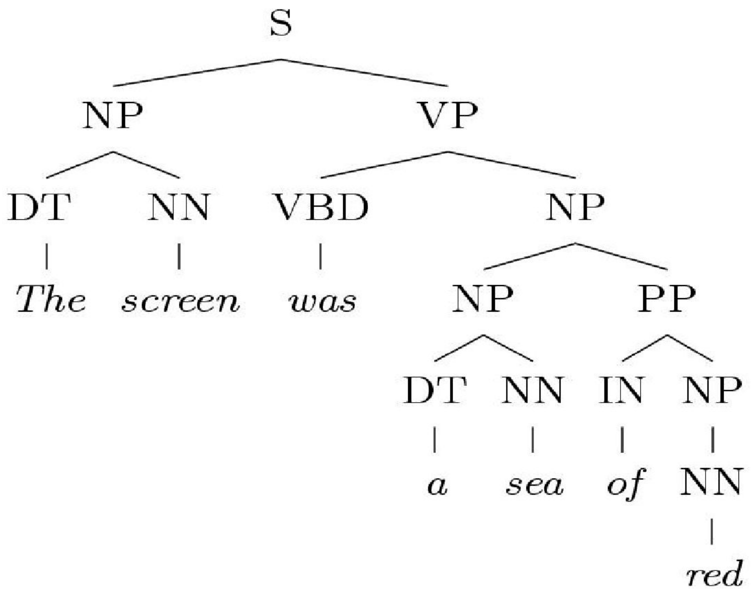
Classification

Sentence → Parse Tree

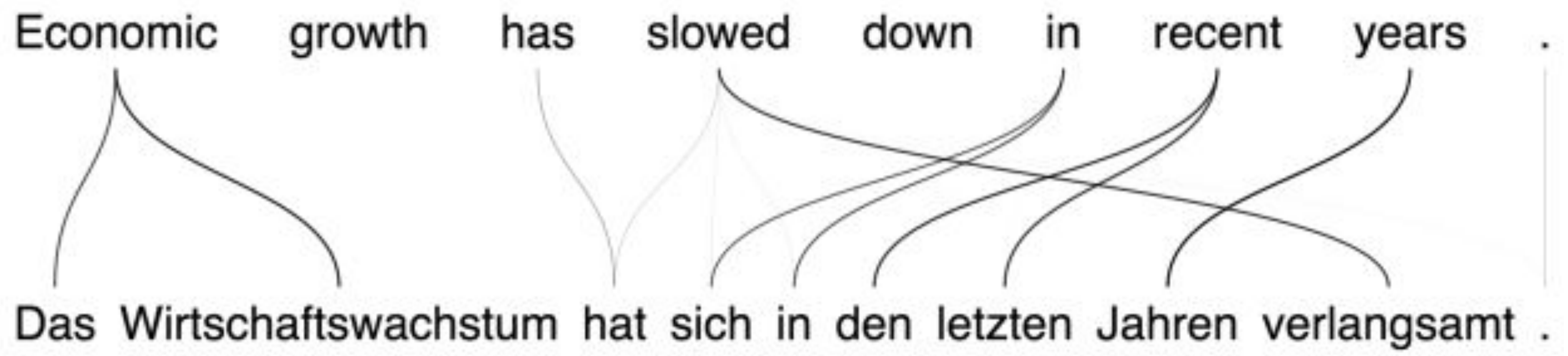
x **y**

The screen was a sea of red



Classification

Sentence ⟶ Translation

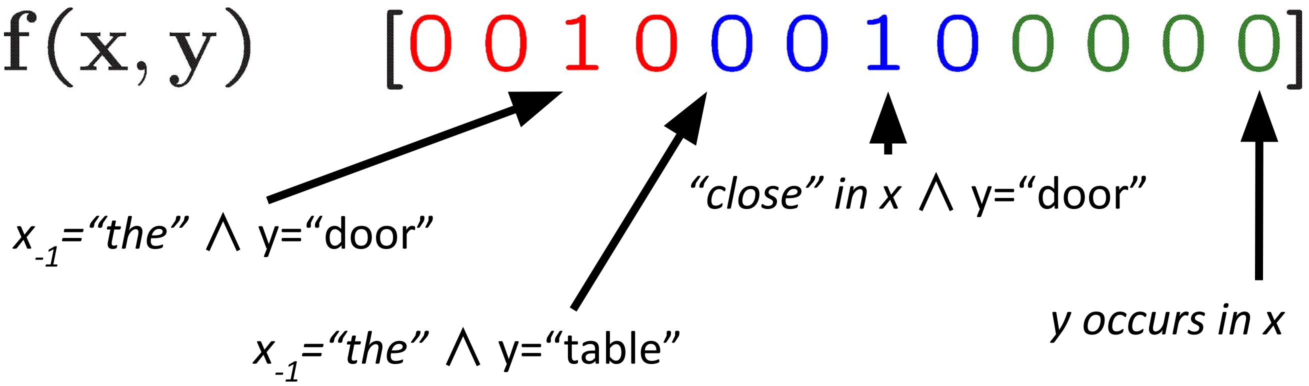


Classification

- Three main ideas
    - Representation as feature vectors
    - Scoring by linear functions
    - Learning (the scoring functions) by optimization

Some Definitions

INPUTS	$\mathbf{x}_i$	*close the ____*
CANDIDATE SET	$\mathcal{Y}(\mathbf{x})$	*{table, door, … }*
CANDIDATE	$\mathbf{y}$	*table*
TRUE OUTPUT	$\mathbf{y}_i^*$	*door*
FEATURE VECTORS	$\mathbf{f}(\mathbf{x},\mathbf{y})$	[0 0 1 0 0 0 1 0 0 0 0 0]



x_{-1}=*"the"* ∧ y="door"

x_{-1}=*"the"* ∧ y="table"

"close" in x ∧ y="door"

y occurs in x

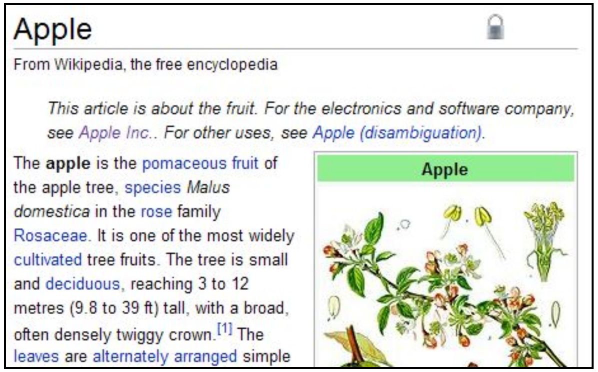
Features

Feature Vectors

- Example: web page ranking (not actually classification)

x_i = “Apple Computers”

$\mathbf{f}_i($



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$) = [0.3\ 5\ 0\ 0\ \ldots]$

$\mathbf{f}_i($

Apple Inc.

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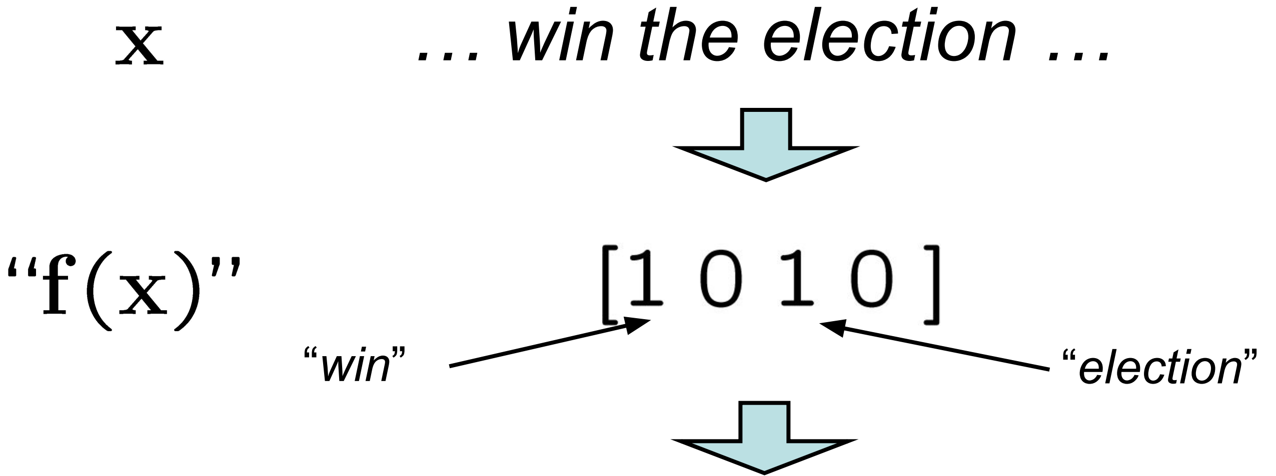
Apple Inc.,

$) = [0.8\ 4\ 2\ 1\ \ldots]$

Block Feature Vectors

- Sometimes, we think of the input as having features, which are multiplied by outputs to form the candidates



$\mathbf{x}$ *… win the election …*

"$\mathbf{f}(\mathbf{x})$" $[1\ 0\ 1\ 0\ ]$

"win" *"election"*

… win the election …

$$\mathbf{f}(SPORTS) = [1\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]$$

… win the election …

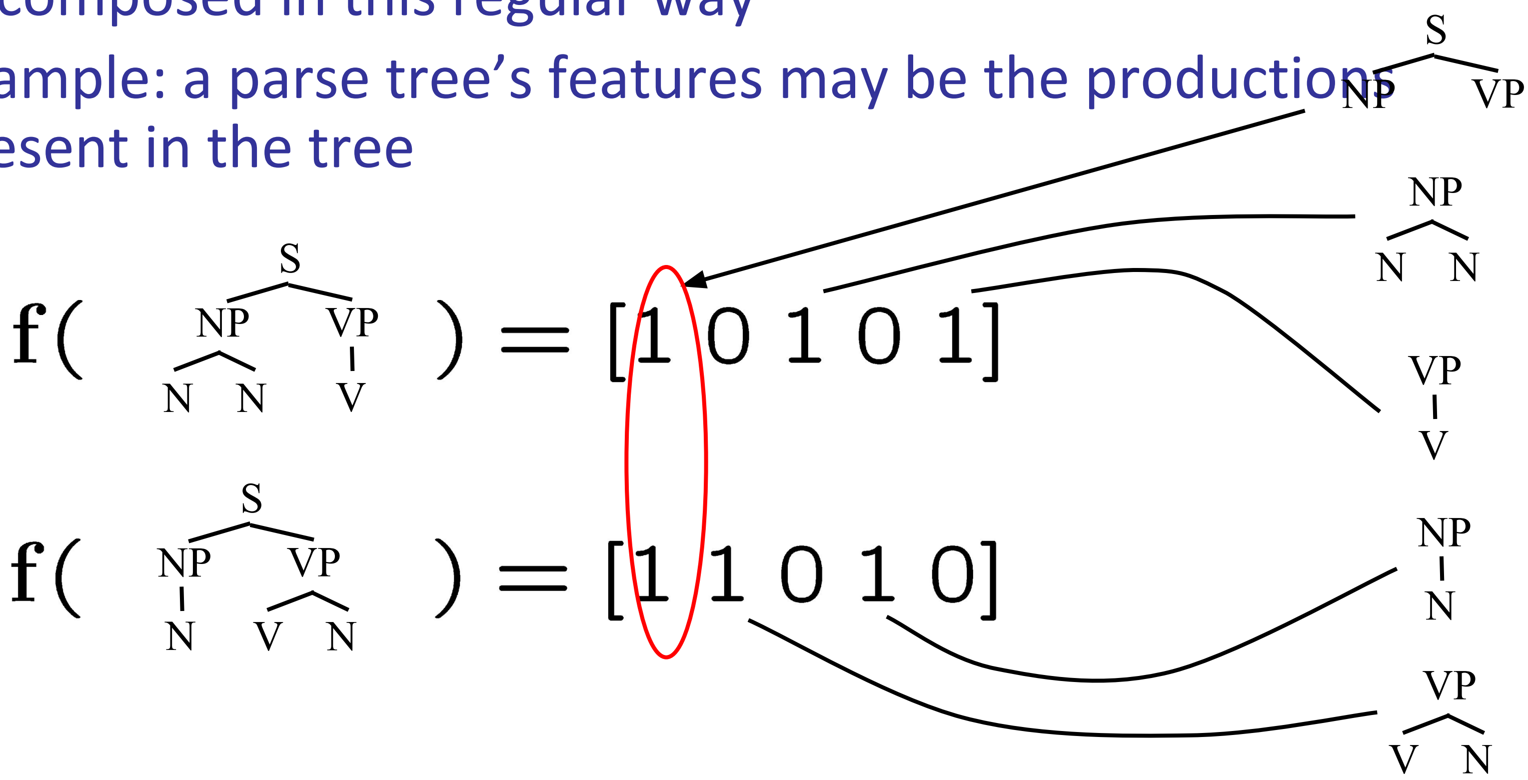
$$\mathbf{f}(POLITICS) = [0\ 0\ 0\ 0\ 1\ 0\ 1\ 0\ 0\ 0\ 0\ 0]$$

… win the election …

$$\mathbf{f}(OTHER) = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 1\ 0]$$

Non-Block Feature Vectors

- Sometimes the features of candidates cannot be decomposed in this regular way
- Example: a parse tree's features may be the productions present in the tree



- Different candidates will thus often share features
- We'll return to the non-block case later

Linear Models

Linear Models: Scoring

- In a linear model, each feature gets a weight w

… win the election …

$$\mathbf{f}(POLITICS) = [\ 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$$

… win the election …

$$\mathbf{f}(SPORTS) = [\ 1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$$

$$\mathbf{w} = [\ 1 \quad 1 \quad -1 \quad -2 \quad 1 \quad -1 \quad 1 \quad -2 \quad -2 \quad -1 \quad -1 \quad 1]$$

- We score hypotheses by multiplying features and weights:

$$score(\mathbf{y}, \mathbf{w}) = \mathbf{w}^\top \mathbf{f}(\mathbf{y})$$

… win the election …

$$\mathbf{f}(POLITICS) = [\ 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$$

$$\mathbf{w} = [\ 1 \quad 1 \quad -1 \quad -2 \quad 1 \quad -1 \quad 1 \quad -2 \quad -2 \quad -1 \quad -1 \quad 1]$$

… win the election …

$$score(POLITICS, \mathbf{w}) = 1 \times 1 + 1 \times 1 = 2$$

Linear Models: Decision Rule

- The linear decision rule:

$$prediction(\text{... win the election ...}, \mathbf{w}) = \arg\max_{\mathbf{y} \in \mathcal{Y}(\mathbf{x})} \mathbf{w}^\top \mathbf{f}(\mathbf{y})$$

$$score(\overset{\text{... win the election ...}}{SPORTS}, \mathbf{w}) = 1 \times 1 + (-1) \times 1 = 0$$

$$score(\overset{\text{... win the election ...}}{POLITICS}, \mathbf{w}) = 1 \times 1 + 1 \times 1 = 2$$

$$score(\overset{\text{... win the election ...}}{OTHER}, \mathbf{w}) = (-2) \times 1 + (-1) \times 1 = -3$$

⇓

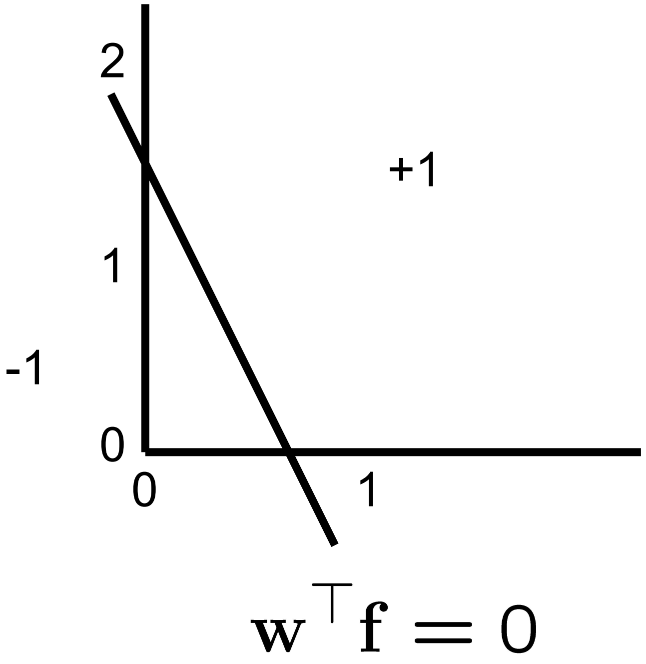
$$prediction(\text{... win the election ...}, \mathbf{w}) = \overset{\text{... win the election ...}}{POLITICS}$$

- We’ve said nothing about where weights come from

Binary Classification

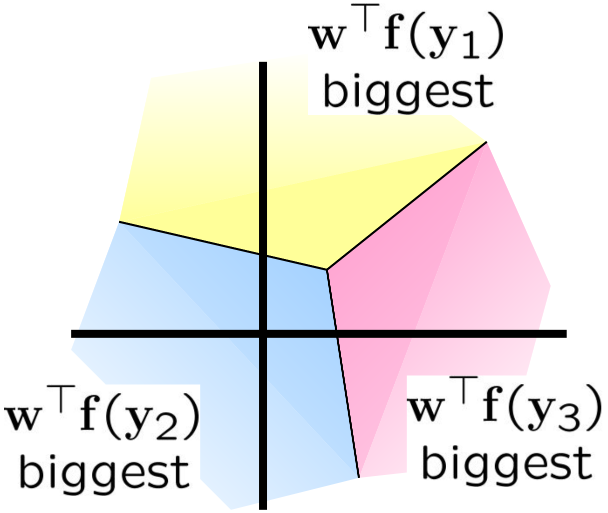
- Important special case: binary classification
  - Classes are y=+1/-1
  - Decision boundary is a hyperplane

    $$\mathbf{w}^\top \mathbf{f}(\mathbf{x}) = 0$$



Multiclass Decision Rule

- If more than two classes:
    - Highest score wins
    - Boundaries are more complex
    - Harder to visualize



$$prediction(\mathbf{x}_i, \mathbf{w}) = \arg\max_{\mathbf{y}\in\mathcal{Y}} \mathbf{w}^\top \mathbf{f}_i(\mathbf{y})$$

Learning

Learning Classifier Weights

- Two broad approaches to learning weights

- Generative: work with a probabilistic model of the data, weights are (log) local conditional probabilities
  - Advantages: learning weights is easy, smoothing is well-understood, backed by understanding of modeling

- Discriminative: set weights based on some error-related criterion
  - Advantages: error-driven, often weights which are good for classification aren't the ones which best describe the data

- We'll mainly talk about the latter for now

How to pick weights?

- Goal: choose “best” vector w given training data
  - For now, we mean “best for classification”

- The ideal: the weights which have greatest test set accuracy / F1 / whatever
  - But, don’t have the test set
  - Must compute weights from training set

- Maybe we want weights which give best training set accuracy?

Minimize Training Error?

- A loss function declares how costly each mistake is

$$\ell_i(\mathbf{y}) = \ell(\mathbf{y}, \mathbf{y}_i^*)$$

  - E.g. 0 loss for correct label, 1 loss for wrong label
  - Can weight mistakes differently (e.g. false positives worse than false negatives or Hamming distance over structured labels)

- We could, in principle, minimize training loss:

$$\min_{\mathbf{w}} \sum_i \ell_i \left(\arg\max_{\mathbf{y}} \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) \right)$$

- This is a hard, discontinuous optimization problem

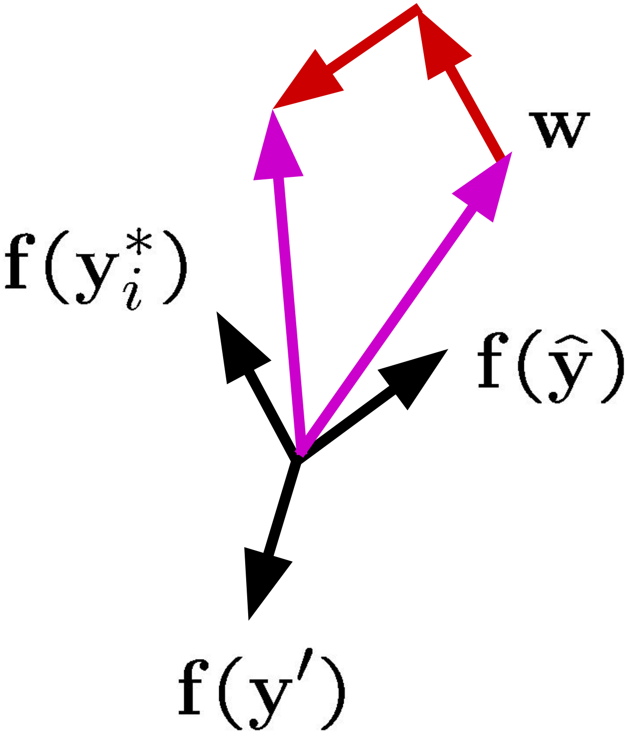
Linear Models: Perceptron

- The perceptron algorithm
  - Iteratively processes the training set, reacting to training errors
  - Can be thought of as trying to drive down training error
- The (online) perceptron algorithm:
  - Start with zero weights w
  - Visit training instances one by one
    - Try to classify

$$\hat{\mathbf{y}} = \arg\max_{\mathbf{y} \in \mathcal{Y}(\mathbf{x})} \mathbf{w}^\top \mathbf{f}(\mathbf{y})$$

    - If correct, no change!
    - If wrong: adjust weights

$$\mathbf{w} \leftarrow \mathbf{w} + \mathbf{f}(\mathbf{y}_i^*)$$
$$\mathbf{w} \leftarrow \mathbf{w} - \mathbf{f}(\hat{\mathbf{y}})$$



Example: “Best” Web Page

$$\mathbf{w} = [1 \quad 2 \quad 0 \quad 0 \quad \ldots]$$

x_i = “Apple Computers”

$\mathbf{f}_i($

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Apple

$) = [0.3\ 5\ 0\ 0\ \ldots]$

$$\mathbf{w}^\top \mathbf{f} = 10.3 \qquad \hat{\mathbf{y}}$$

$\mathbf{f}_i($

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Apple Inc.,

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$) = [0.8\ 4\ 2\ 1\ \ldots]$

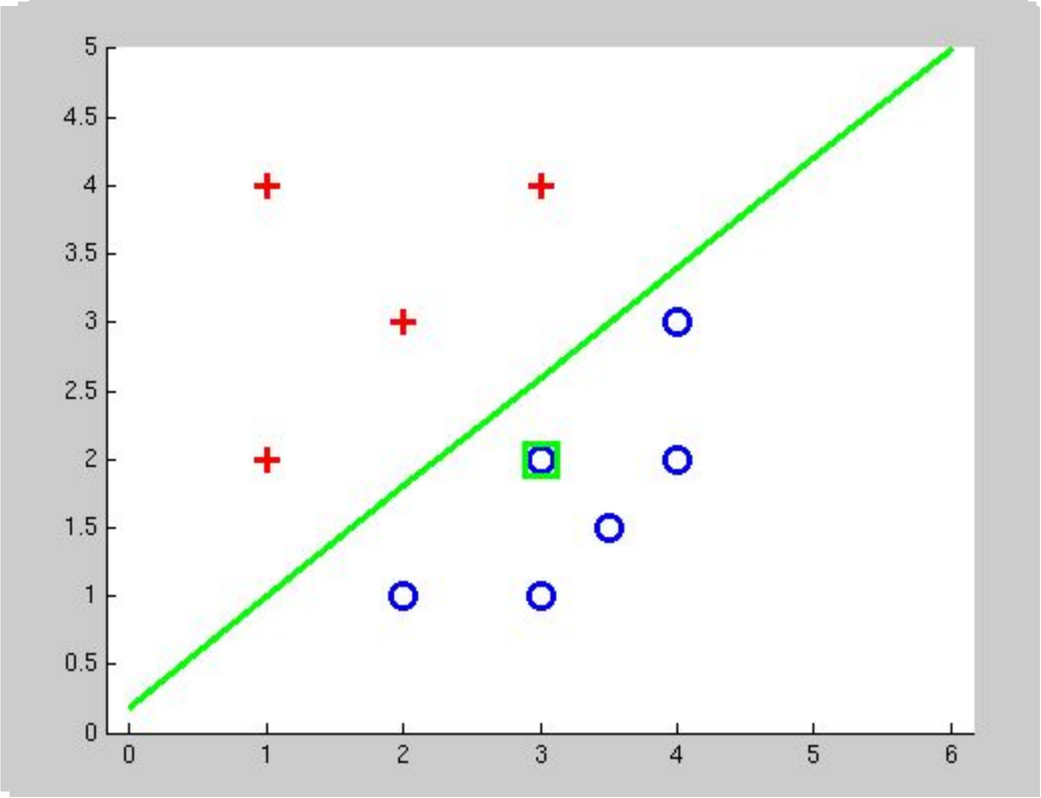
$$\mathbf{w}^\top \mathbf{f} = 8.8 \qquad \mathbf{y}_i^*$$

$$\mathbf{w} \leftarrow \mathbf{w} + \mathbf{f}(\mathbf{y}_i^*) - \mathbf{f}(\hat{\mathbf{y}})$$

$$\mathbf{w} = [1.5 \quad 1 \quad 2 \quad 1 \quad \ldots]$$

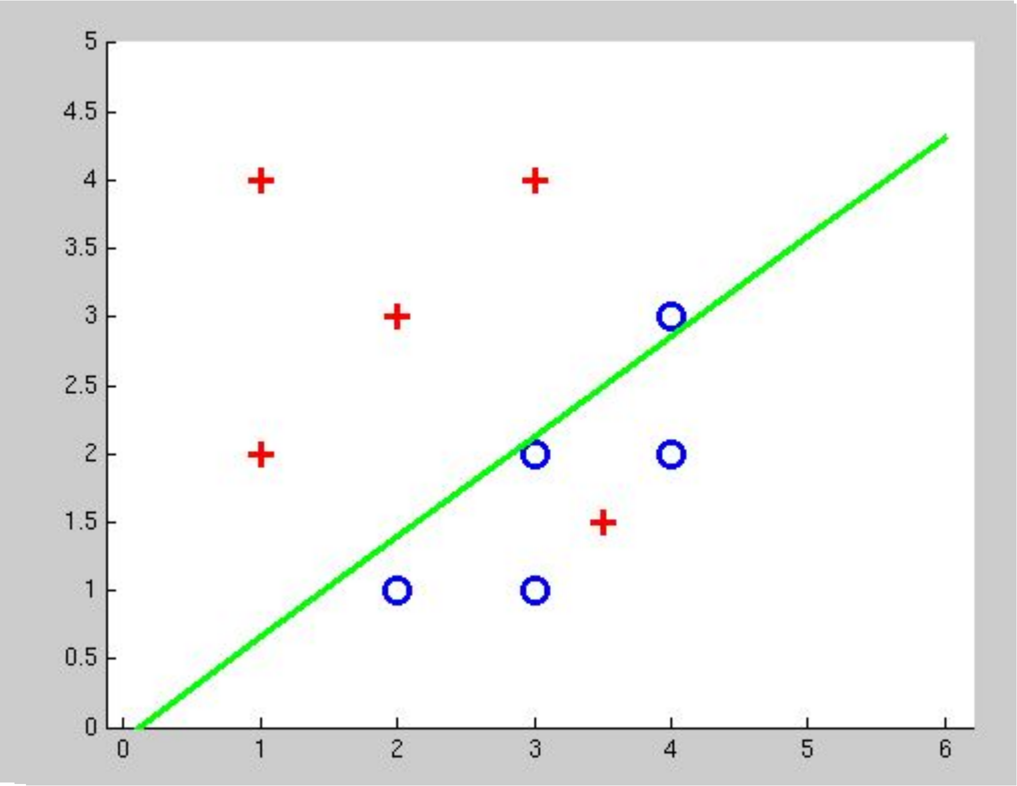
Examples: Perceptron

- Separable Case



Examples: Perceptron

- Non-Separable Case

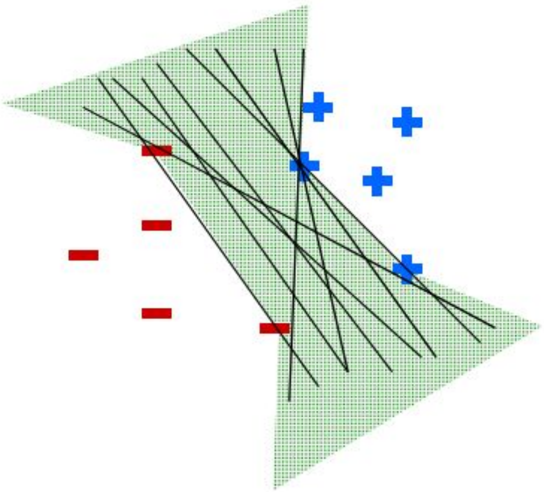


Problems with Perceptron

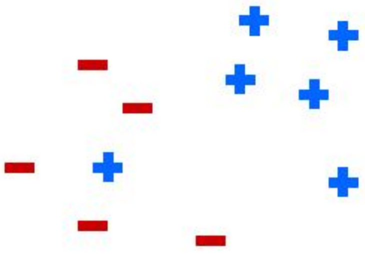
- Perceptron “Goal”: Seperate the training data

$$\forall i, \forall \mathbf{y} \neq \mathbf{y}^i \quad \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}^i) \geq \mathbf{w}^\top \mathbf{f}_i(\mathbf{y})$$

1. This may be an entire feasible space



2. Or it may be impossible



Objective Functions

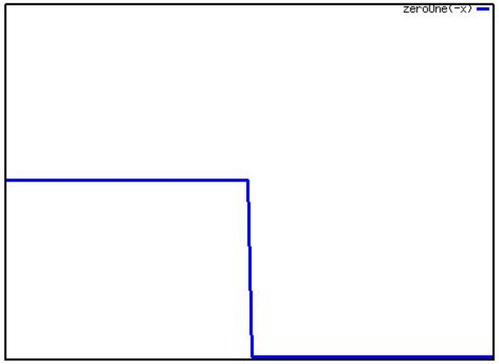
- What do we want from our weights?
  - So far: minimize (training) errors:

$$\min_{\mathbf{w}} \sum_i \ell_i \left(\arg\max_{\mathbf{y}} \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) \right)$$

or

$$\sum_i step \left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) - \max_{\mathbf{y} \neq \mathbf{y}_i^*} \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) \right)$$

  - This is the “zero-one loss”
    - Discontinuous, minimizing is NP-complete
  - Maximum entropy and SVMs have other objectives related to zero-one loss

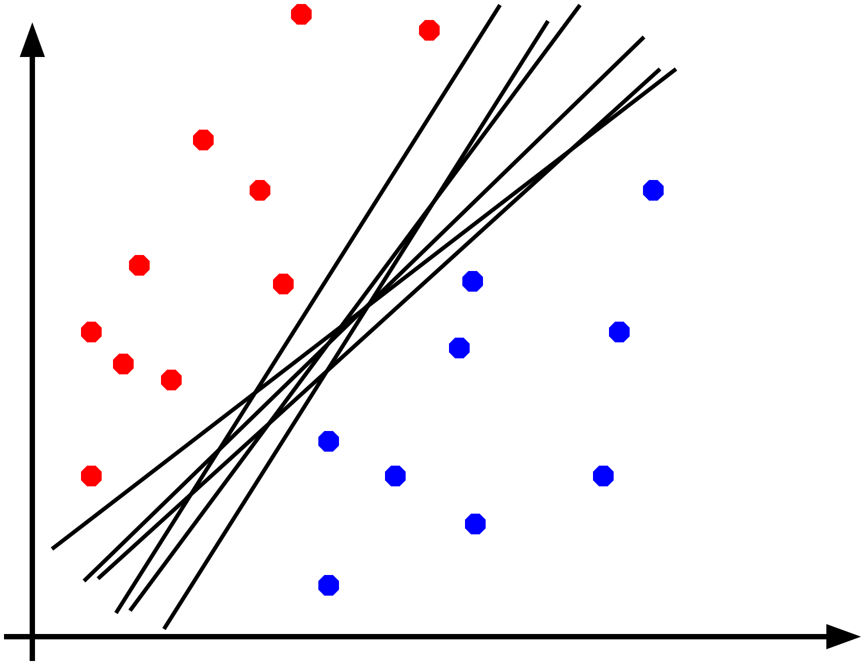


$$\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}^i) - \max_{\mathbf{y} \neq \mathbf{y}_i^*} \mathbf{w}^\top \mathbf{f}_i(\mathbf{y})$$

Margin

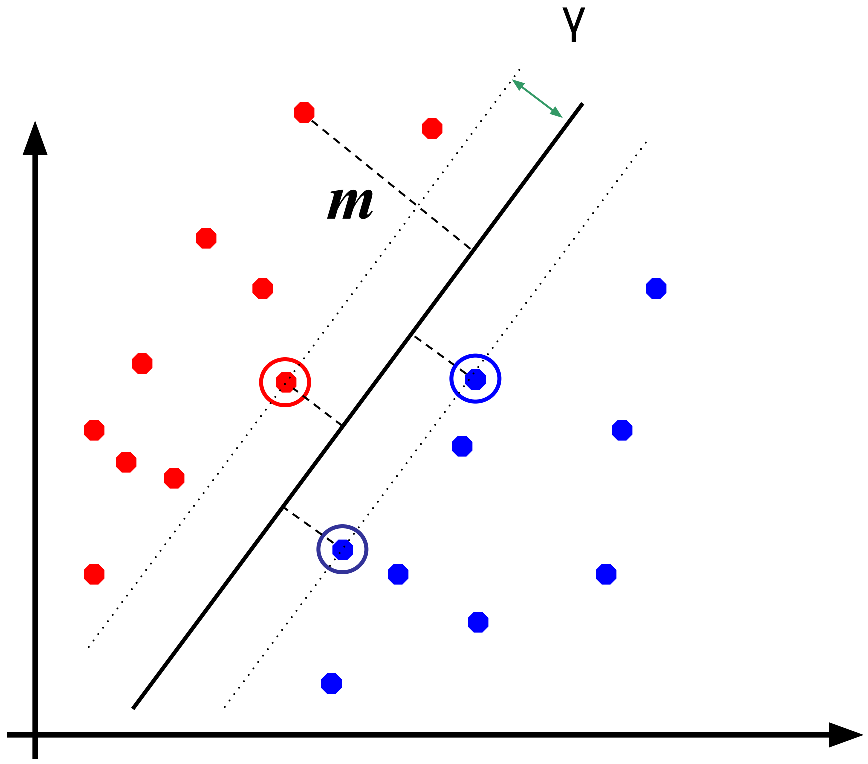
Linear Separators

- Which of these linear separators is optimal?



Classification Margin (Binary)

- Distance of $\mathbf{x}_i$ to separator is its margin, m_i
- Examples closest to the hyperplane are support vectors
- Margin γ of the separator is the minimum m



Classification Margin

- For each example $\mathbf{x}_i$ and possible mistaken candidate $\mathbf{y}$, we avoid that mistake by a margin $m_i(\mathbf{y})$ (with zero-one loss)

$$m_i(\mathbf{y}) = \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) - \mathbf{w}^\top \mathbf{f}_i(\mathbf{y})$$

- Margin γ of the entire separator is the minimum *m*

$$\gamma = \min_i \left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) - \max_{\mathbf{y} \neq \mathbf{y}_i^*} \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) \right)$$

- It is also the largest γ for which the following constraints hold

$$\forall i, \forall \mathbf{y} \quad \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) \geq \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) + \gamma \ell_i(\mathbf{y})$$

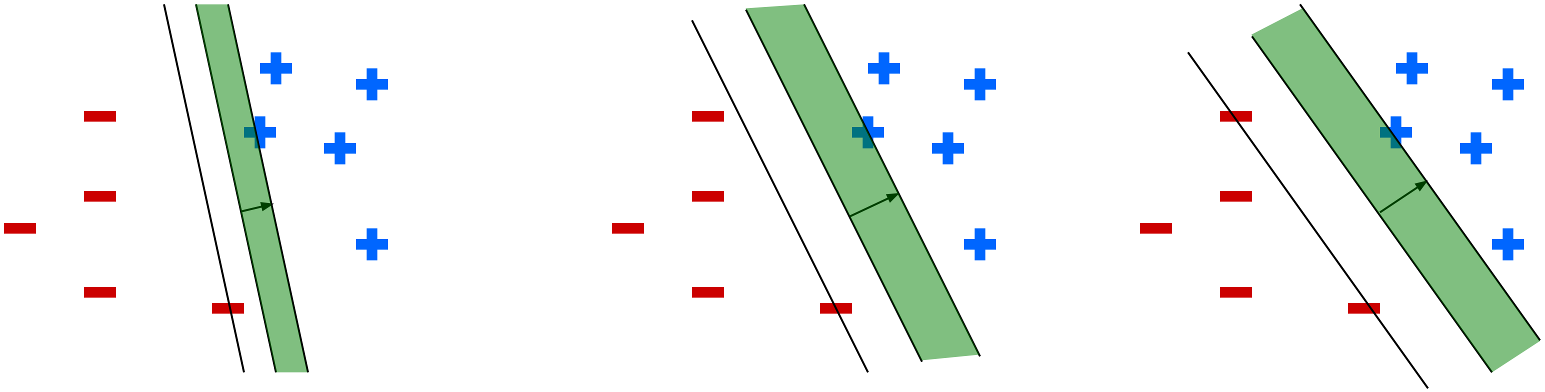
Maximum Margin

- Separable SVMs: find the max-margin w

$$\max_{||\mathbf{w}||=1} \gamma$$

$$\ell_i(\mathbf{y}) = \begin{cases} 0 & \text{if } \mathbf{y} = \mathbf{y}_i^* \\ 1 & \text{if } \mathbf{y} \neq \mathbf{y}_i^* \end{cases}$$

$$\forall i, \forall \mathbf{y} \quad \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) \geq \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) + \gamma \ell_i(\mathbf{y})$$



- Can stick this into Matlab and (slowly) get an SVM
- Won't work (well) if non-separable

Max Margin / Small Norm

- Reformulation: find the smallest w which separates data

Remember this condition?

$$\max_{||\mathbf{w}||=1} \gamma$$

$$\forall i, \mathbf{y} \quad \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) \geq \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) + \gamma \ell_i(\mathbf{y})$$

- γ scales linearly in w, so if ||w|| isn't constrained, we can take any separating w and scale up our margin

$$\gamma \quad = \quad \min_{i, \mathbf{y} \neq \mathbf{y}_i^*} [\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) - \mathbf{w}^\top \mathbf{f}_i(\mathbf{y})] / \ell_i(\mathbf{y})$$

- Instead of fixing the scale of w, we can fix $\gamma = 1$

$$\min_{\mathbf{w}} \frac{1}{2} ||\mathbf{w}||^2$$

$$\forall i, \mathbf{y} \quad \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) \geq \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) + 1 \ell_i(\mathbf{y})$$

Gamma to w

$$\max_{||\mathbf{w}||=1} \gamma$$

$$\forall i, \mathbf{y} \quad \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) \geq \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) + \gamma \ell_i(\mathbf{y})$$

$$\mathbf{w} = \gamma u$$

$$\gamma = 1/||u||$$

$$\max_{||\gamma u||=1} 1/||u||^2$$

$$\forall i, \mathbf{y} \quad \gamma u^\top \mathbf{f}_i(\mathbf{y}_i^*) \geq \gamma u^\top \mathbf{f}_i(\mathbf{y}) + \gamma \ell_i(\mathbf{y})$$

$$\max_{||\gamma u||=1} 1/||u||^2$$

$$\forall i, \mathbf{y} \quad u^\top \mathbf{f}_i(\mathbf{y}_i^*) \geq u^\top \mathbf{f}_i(\mathbf{y}) + \ell_i(\mathbf{y})$$

$$\min_{||\gamma u||=1} ||u||^2$$

$$\forall i, \mathbf{y} \quad u^\top \mathbf{f}_i(\mathbf{y}_i^*) \geq u^\top \mathbf{f}_i(\mathbf{y}) + \ell_i(\mathbf{y})$$

$$\min_u ||u||^2$$

$$\forall i, \mathbf{y} \quad u^\top \mathbf{f}_i(\mathbf{y}_i^*) \geq u^\top \mathbf{f}_i(\mathbf{y}) + \ell_i(\mathbf{y})$$

$$\min_u \frac{1}{2}||u||^2$$

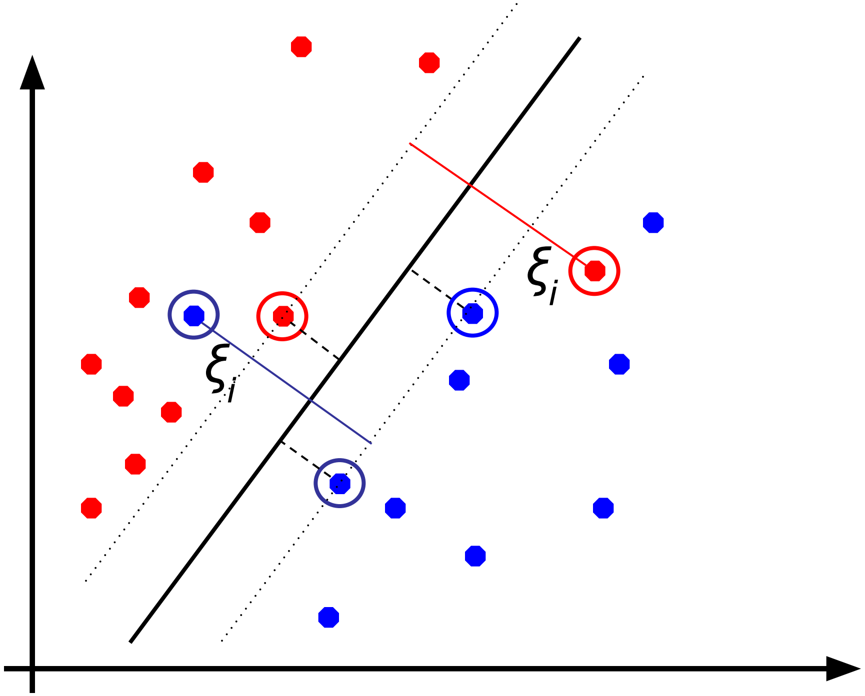
$$\forall i, \mathbf{y} \quad u^\top \mathbf{f}_i(\mathbf{y}_i^*) \geq u^\top \mathbf{f}_i(\mathbf{y}) + \ell_i(\mathbf{y})$$

$$\min_{\mathbf{w}} \frac{1}{2}||\mathbf{w}||^2$$

$$\forall i, \mathbf{y} \quad \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) \geq \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) + \ell_i(\mathbf{y})$$

Soft Margin Classification

- What if the training set is not linearly separable?
- *Slack variables* ξ_i can be added to allow misclassification of difficult or noisy examples, resulting in a *soft margin* classifier



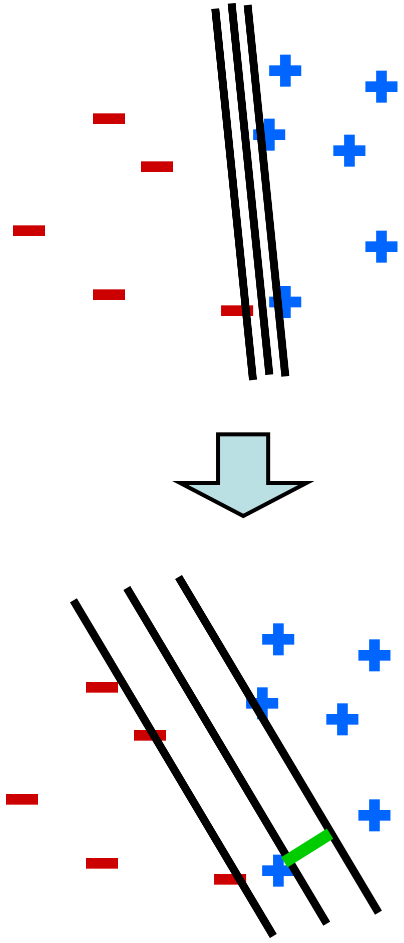
Maximum Margin

Note: exist other choices of how to penalize slacks!

- Non-separable SVMs
    - Add slack to the constraints
    - Make objective pay (linearly) for slack:

$$\min_{\mathbf{w},\xi} \frac{1}{2}||\mathbf{w}||^2 + C \sum_i \xi_i$$

$$\forall i, \mathbf{y}, \quad \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) + \xi_i \geq \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) + \ell_i(\mathbf{y})$$



-
    - C is called the *capacity* of the SVM – the smoothing knob
- Learning:
    - Can still stick this into Matlab if you want
    - Constrained optimization is hard; better methods!

Hinge Loss

- We have a constrained minimization

$$\min_{\mathbf{w},\xi} \frac{1}{2}||\mathbf{w}||^2 + C\sum_i \xi_i$$

$$\forall i, \mathbf{y}, \quad \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) + \xi_i \geq \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) + \ell_i(\mathbf{y})$$

- …but we can solve for ξ_i

$$\forall i, \mathbf{y}, \quad \xi_i \geq \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) + \ell_i(\mathbf{y}) - \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*)$$

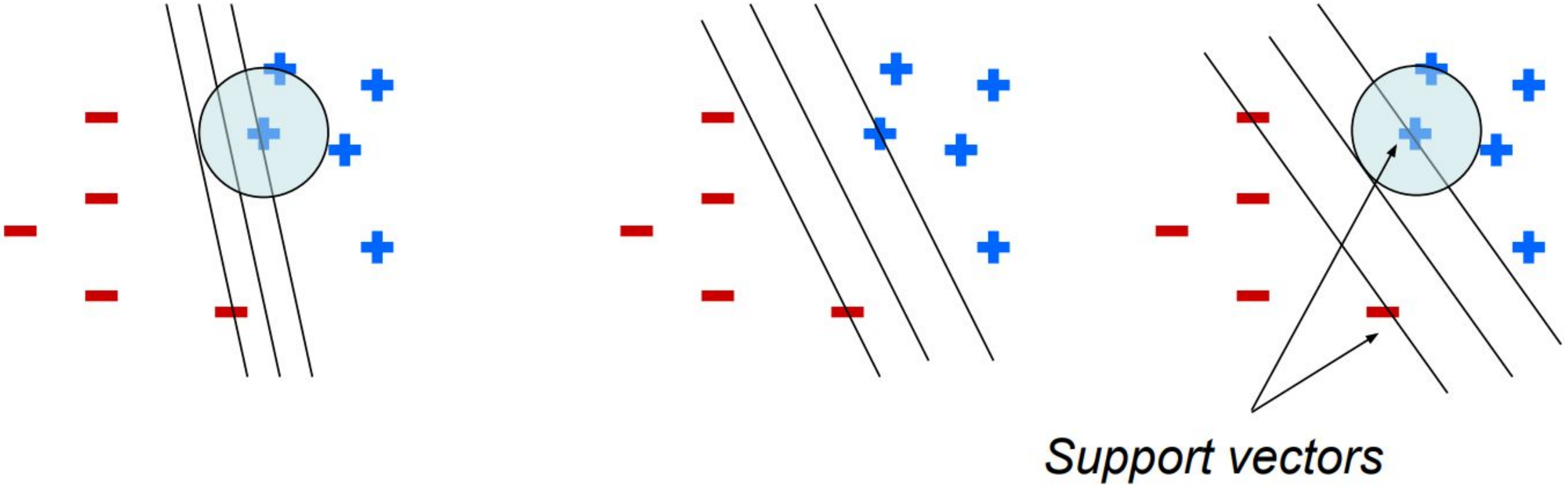
$$\forall i, \quad \xi_i = \max_{\mathbf{y}} \left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) + \ell_i(\mathbf{y})\right) - \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*)$$

- Giving

$$\min_{\mathbf{w}} \frac{1}{2}||\mathbf{w}||^2 + C\sum_i \left(\max_{\mathbf{y}} \left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) + \ell_i(\mathbf{y})\right) - \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*)\right)$$

Why Max Margin?

- Why do this? Various arguments:
  - Solution depends only on the boundary cases, or *support vectors*
  - Solution robust to movement of support vectors
  - Sparse solutions (features not in support vectors get zero weight)
  - Generalization bound arguments
  - Works well in practice for many problems



Likelihood

Linear Models: Maximum Entropy

- Maximum entropy (logistic regression)
  - Use the scores as probabilities:

$$\mathsf{P}(\mathbf{y}|\mathbf{x},\mathbf{w}) = \frac{\exp(\mathbf{w}^\top \mathbf{f}(\mathbf{y}))}{\sum_{\mathbf{y}'} \exp(\mathbf{w}^\top \mathbf{f}(\mathbf{y}'))}$$

Make positive

Normalize

  - Maximize the (log) conditional likelihood of training data

$$L(\mathbf{w}) = \log \prod_i \mathsf{P}(\mathbf{y}_i^*|\mathbf{x}_i,\mathbf{w}) = \sum_i \log \left(\frac{\exp(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*))}{\sum_{\mathbf{y}} \exp(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}))} \right)$$

$$= \sum_i \left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) - \log \sum_{\mathbf{y}} \exp(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y})) \right)$$

Maximum Entropy II

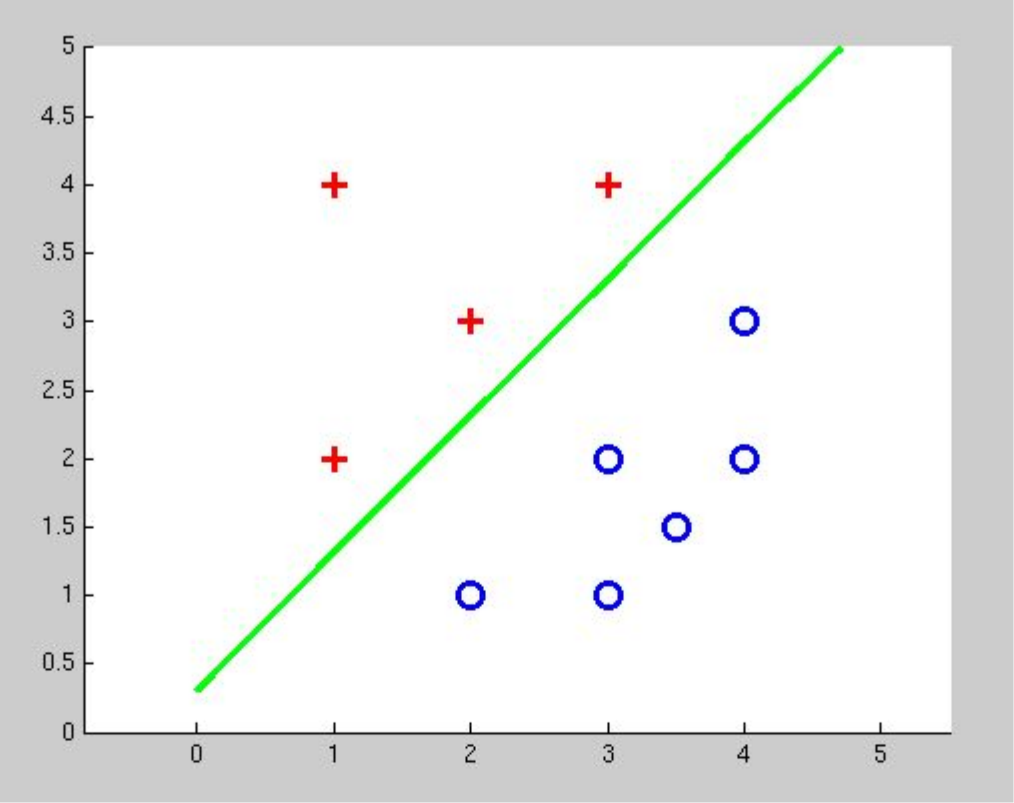
- Motivation for maximum entropy:
  - Connection to maximum entropy principle (sort of)
  - Might want to do a good job of being uncertain on noisy cases…
  - … in practice, though, posteriors are pretty peaked

- Regularization (smoothing)

$$\max_{\mathbf{w}} \sum_i \left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) - \log \sum_{\mathbf{y}} \exp(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y})) \right) - k||\mathbf{w}||^2$$

$$\min_{\mathbf{w}} \; k||\mathbf{w}||^2 - \sum_i \left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) - \log \sum_{\mathbf{y}} \exp(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y})) \right)$$

Maximum Entropy



Loss Comparison

Log-Loss

- If we view maxent as a minimization problem:

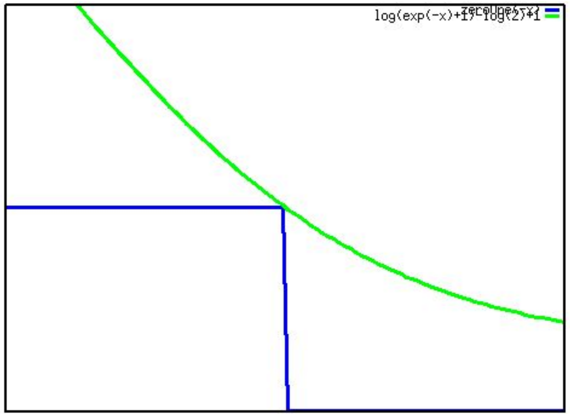
$$\min_{\mathbf{w}} \; k||\mathbf{w}||^2 + \sum_i - \left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) - \log \sum_{\mathbf{y}} \exp(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y})) \right)$$

- This minimizes the "log loss" on each example

$$- \left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) - \log \sum_{\mathbf{y}} \exp(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y})) \right)$$

$$step\left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) - \max_{\mathbf{y} \neq \mathbf{y}_i^*} \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) \right)$$

- Or *d* on z



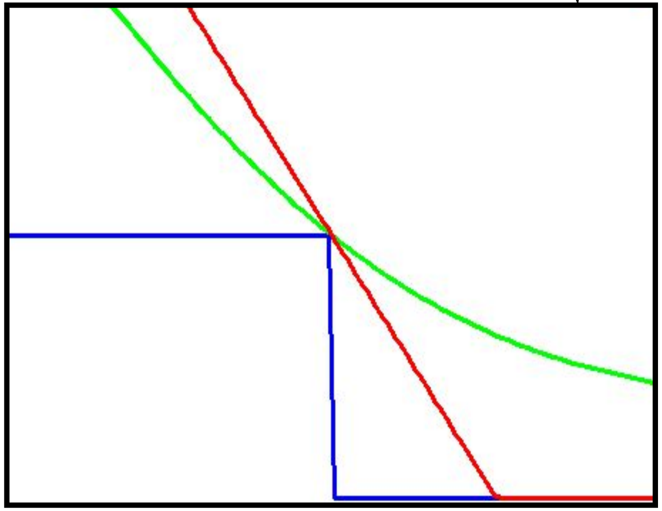
Remember SVMs - Hinge Loss

- Consider the per-instance objective:

$$\min_{\mathbf{w}} \; k||\mathbf{w}||^2 + \sum_i \left(\max_{\mathbf{y}} \left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) + \ell_i(y) \right) - \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) \right)$$

Plot really only right in binary case

- This is called the "hinge loss"
    - Unlike maxent / log loss, you stop gaining objective once the true label wins by enough
    - You can start from here and derive the SVM objective
    - Can solve directly with sub-gradient decent (e.g. Pegasos: Shalev-Shwartz et al 07)



$$\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) - \max_{\mathbf{y} \neq \mathbf{y}_i^*} \left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) \right)$$

Max vs “Soft-Max” Margin

- SVMs:

$$\min_{\mathbf{w}} \quad k||\mathbf{w}||^2 - \sum_i \left(\underbrace{\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) - \max_{\mathbf{y}} \left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) + \ell_i(y) \right)}_{\text{You can make this zero}} \right)$$

- Maxent:

$$\min_{\mathbf{w}} \quad k||\mathbf{w}||^2 - \sum_i \left(\underbrace{\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) - \log \sum_{\mathbf{y}} \exp \left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) \right)}_{\text{... but not this one}} \right)$$

- Ver, better than a function of the other scores
  - The SVM tries to beat the augmented runner-up
  - The Maxent classifier tries to beat the “soft-max”

Loss Functions: Comparison

- Zero-One Loss

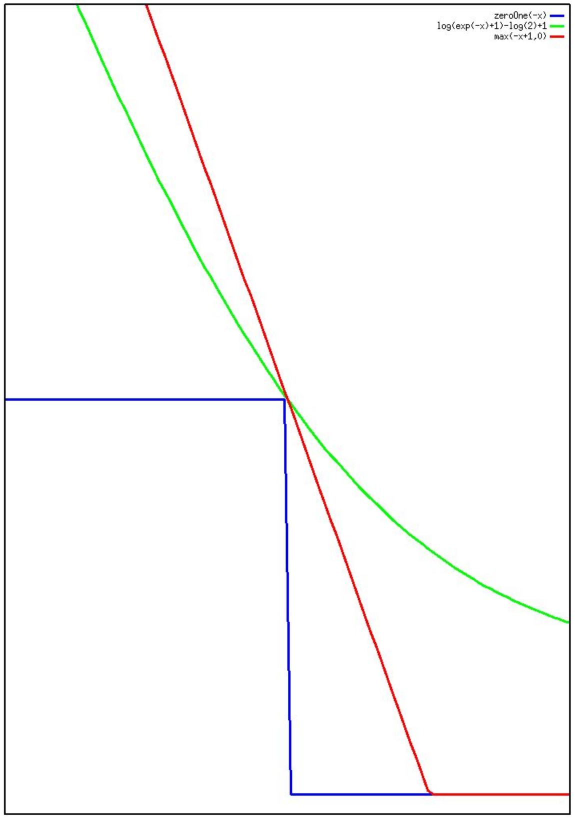
$$\sum_i \mathit{step}\left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) - \max_{\mathbf{y} \neq \mathbf{y}_i^*} \mathbf{w}^\top \mathbf{f}_i(\mathbf{y})\right)$$

- Hinge

$$\sum_i \left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) - \max_{\mathbf{y}} \left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) + \ell_i(y)\right)\right)$$

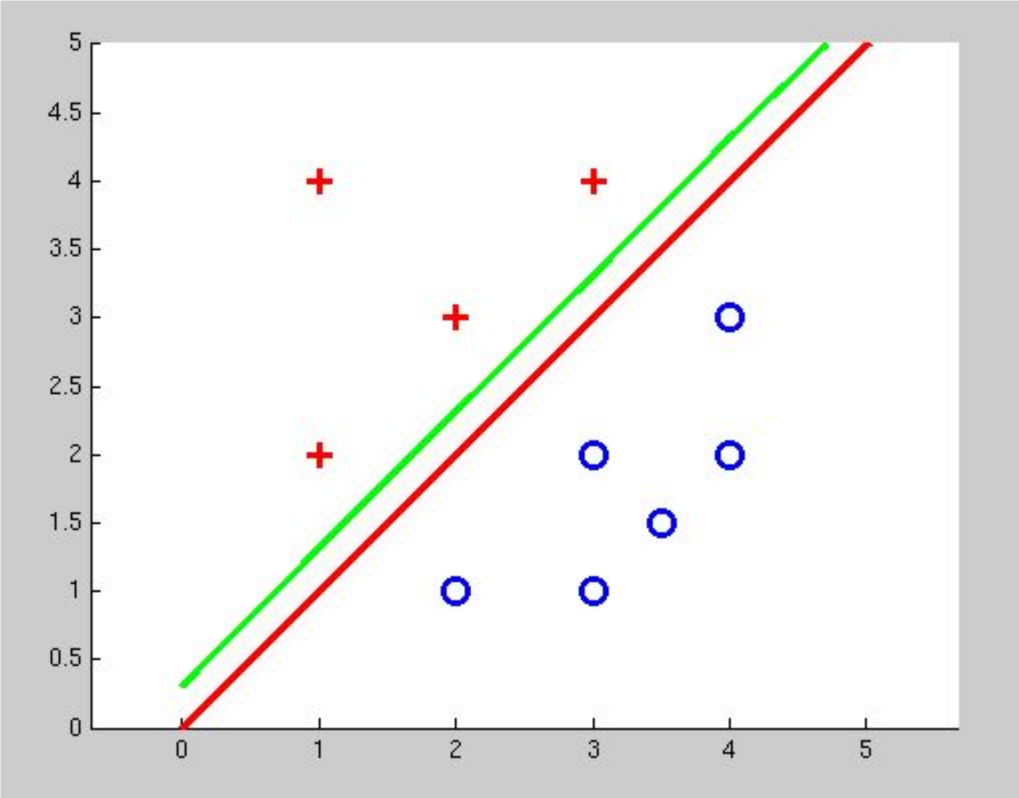
- Log

$$\sum_i \left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) - \log \sum_{\mathbf{y}} \exp\left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y})\right)\right)$$



$$\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) - \max_{\mathbf{y} \neq \mathbf{y}_i^*} \left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y})\right)$$

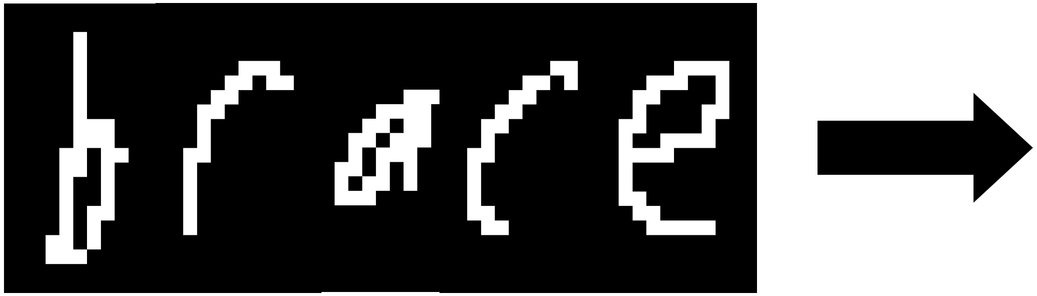
Separators: Comparison



Structure

Handwriting recognition

x **y**



brace

Sequential structure

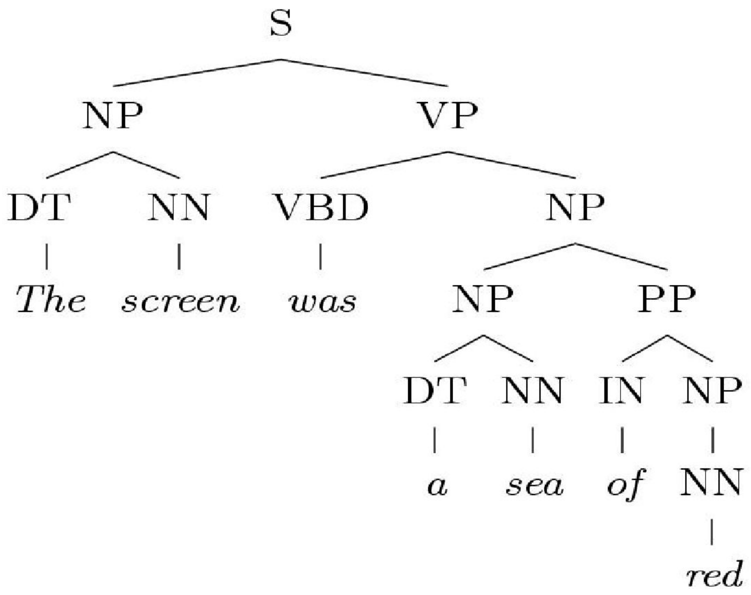
[Slides: Taskar and Klein 05]

CFG Parsing

x

y

The screen was a sea of red



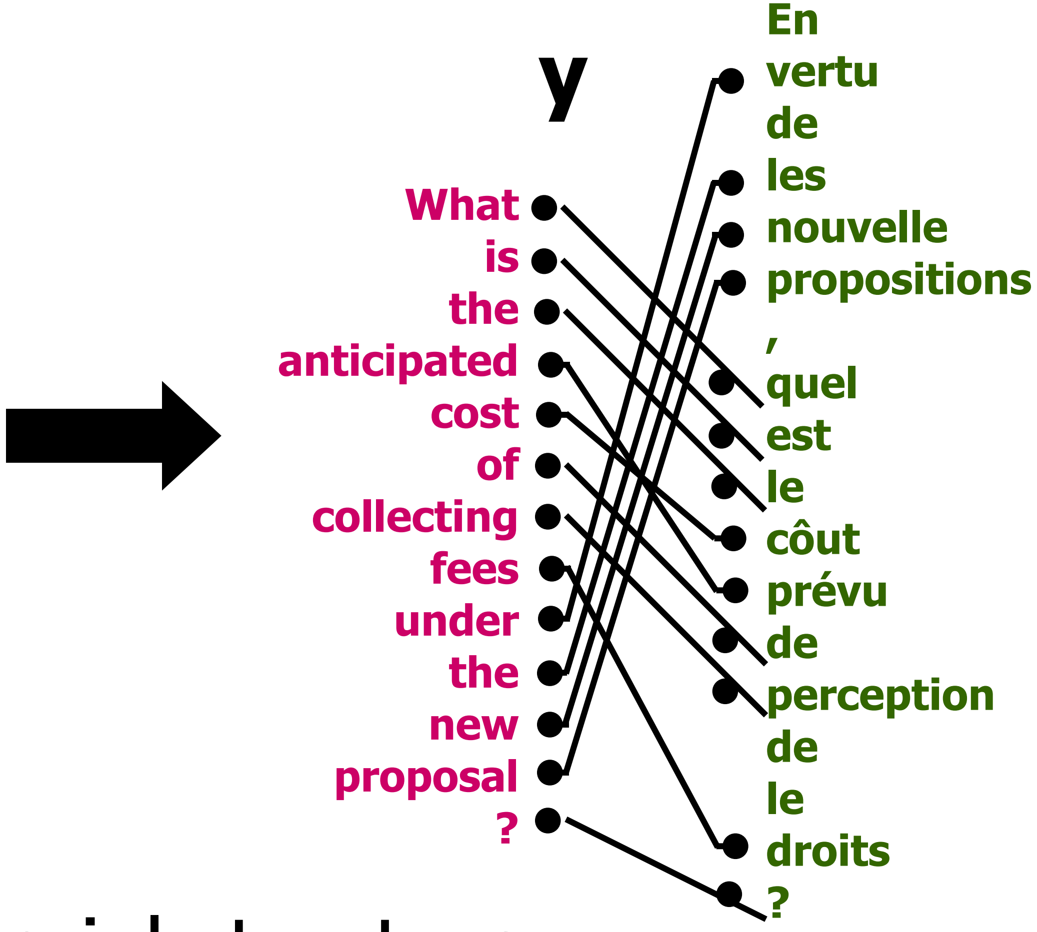
Recursive structure

Bilingual Word Alignment

x

What is the anticipated cost of collecting fees under the new proposal?

En vertu de nouvelle propositions, quel est le côut prévu de perception de les droits?



y

What
is
the
anticipated
cost
of
collecting
fees
under
the
new
proposal
?

En
vertu
de
les
nouvelle
propositions
,
quel
est
le
côut
prévu
de
perception
de
le
droits
?

Combinatorial structure

Definitions

INPUTS	$\mathbf{x}_i$
CANDIDATE SET	$\mathcal{Y}(\mathbf{x})$
CANDIDATES	$\mathbf{y}$
TRUE OUTPUTS	$\mathbf{y}_i^*$
FEATURE VECTORS	$\mathbf{f}(\mathbf{x}, \mathbf{y})$

Structured Models

$$\mathit{prediction}(\mathbf{x},\mathbf{w}) = \operatorname*{arg\,max}_{\mathbf{y}\in\mathcal{Y}(\mathbf{x})} \mathit{score}(\mathbf{y},\mathbf{w})$$

space of feasible outputs

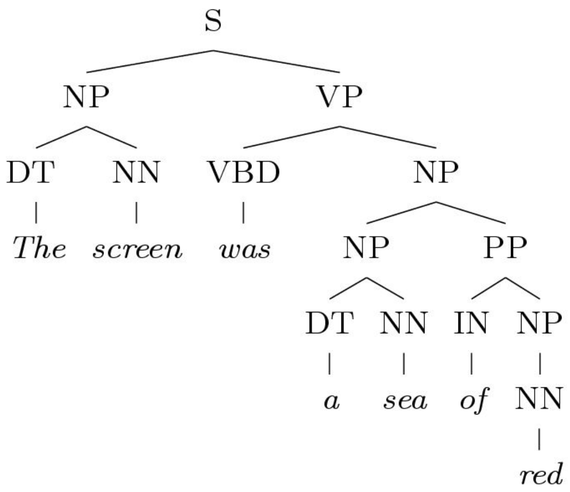
Assumption:

$$\mathit{score}(\mathbf{y},\mathbf{w}) = \mathbf{w}^\top \mathbf{f}(\mathbf{y}) = \sum_p \mathbf{w}^\top \mathbf{f}(\mathbf{y}_p)$$

Score is a sum of local “part” scores

Parts = nodes, edges, productions

CFG Parsing



$$\mathbf{f} : \mathcal{X} \times \mathcal{Y} \rightarrow \Re^d$$

#(NP → DT NN)

…

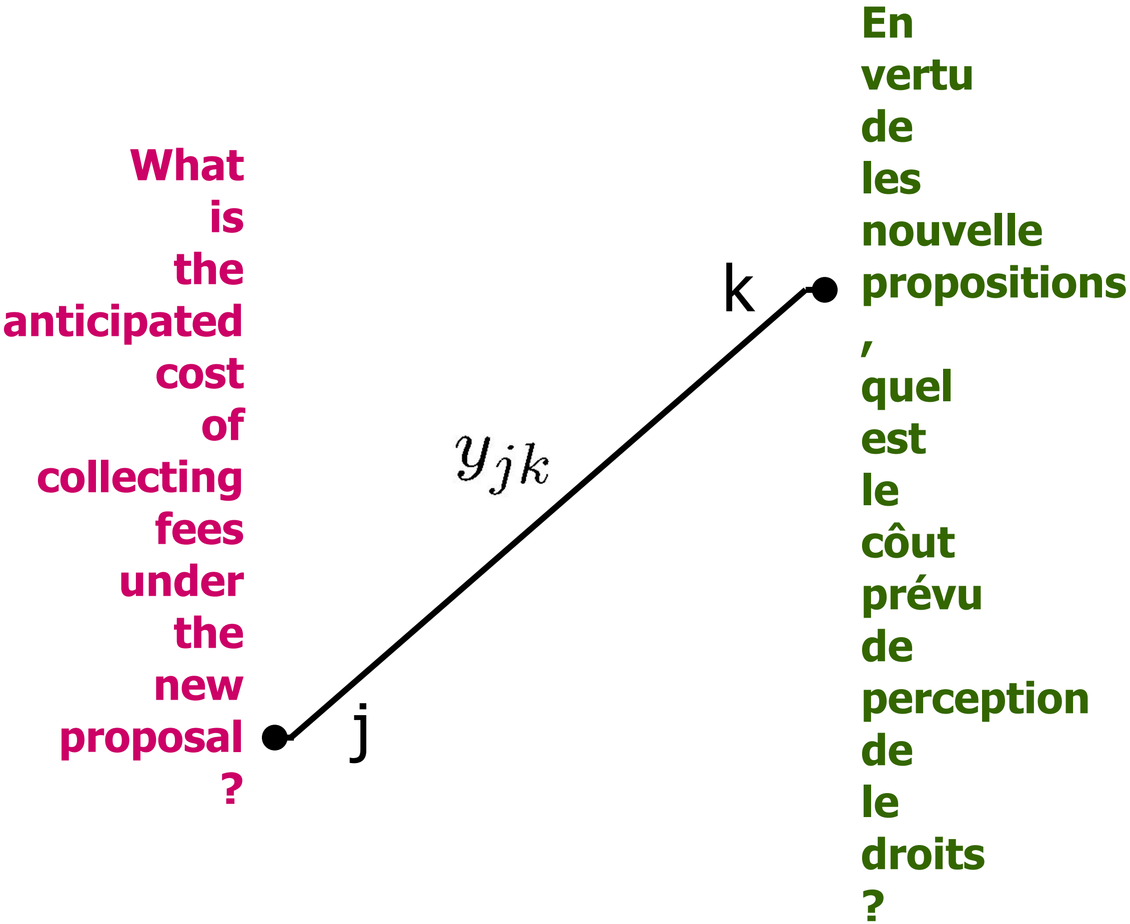
#(PP → IN NP)

…

#(NN → ‘sea’)

Bilingual word alignment

$$\sum_{y_{jk} \in \mathbf{y}} \mathbf{w}^\top \mathbf{f}(\mathbf{x}_{jk}) = \mathbf{w}^\top \mathbf{f}(\mathbf{x}, \mathbf{y})$$



What
is
the
anticipated
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En
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de
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nouvelle
propositions
,
quel
est
le
côut
prévu
de
perception
de
le
droits
?

$\mathbf{f}(\mathbf{x}_{jk})$

- association
- position
- orthography

Efficient Decoding

- Common case: you have a black box which computes

$$\text{prediction}(\mathbf{x}) = \underset{\mathbf{y} \in \mathcal{Y}(\mathbf{x})}{\arg\max}\, \mathbf{w}^\top \mathbf{f}(\mathbf{y})$$

at least approximately, and you want to learn w

- Easiest option is the structured perceptron [Collins 01]
  - Structure enters here in that the search for the best y is typically a combinatorial algorithm (dynamic programming, matchings, ILPs, A*…)
  - Prediction is structured, learning update is not

Structured Margin (Primal)

Remember our primal margin objective?

$$\min_{w} \quad \frac{1}{2}\|w\|_2^2 + C\sum_i \left(\max_y \left(w^\top f_i(y) + \ell_i(y)\right) - w^\top f_i(y_i^*)\right)$$

Still applies with structured output space!

Structured Margin (Primal)

Just need efficient loss-augmented decode:

$$\bar{y} = \operatorname{argmax}_y \left(w^\top f_i(y) + \ell_i(y) \right)$$

$$\min_w \quad \frac{1}{2}\|w\|_2^2 + C \sum_i \left(w^\top f_i(\bar{y}) + \ell_i(\bar{y}) - w^\top f_i(y_i^*) \right)$$

$$\nabla_w = w + C \sum_i \left(f_i(\bar{y}) - f_i(y_i^*) \right)$$

Still use general subgradient descent methods! (Adagrad)

Structured Margin

- Remember the constrained version of primal:

$$\begin{array}{ll} \min_{\mathbf{w},\xi} & \frac{1}{2}||\mathbf{w}||^2 + C\sum_i \xi_i \\ \forall i, \mathbf{y} & \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) \geq \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) + \ell_i(\mathbf{y}) - \xi_i \end{array}$$

Full Margin: OCR

- We want:

$\arg\max_{\mathbf{y}} \ \mathbf{w}^\top \mathbf{f}($  $, \mathbf{y}) \ = \ $ "brace"

- Equivalently:

$\mathbf{w}^\top \mathbf{f}($  $,$ "brace"$) \ > \ \mathbf{w}^\top \mathbf{f}($  $,$ "aaaaa"$)$

$\mathbf{w}^\top \mathbf{f}($  $,$ "brace"$) \ > \ \mathbf{w}^\top \mathbf{f}($  $,$ "aaaab"$)$

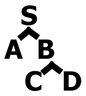
$\ldots$

$\mathbf{w}^\top \mathbf{f}($  $,$ "brace"$) \ > \ \mathbf{w}^\top \mathbf{f}($  $,$ "zzzzz"$)$

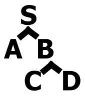
$\Bigg\}$ **a lot!**

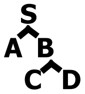
Parsing example

- We want:

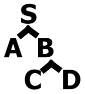
$\arg\max_{\mathrm{y}} \; \mathrm{w}^\top \mathrm{f}($ **'It was red'** $, \mathrm{y}) \; = $ 

- Equivalently:

$\mathrm{w}^\top \mathrm{f}($**'It was red',**  $) \; > \; \mathrm{w}^\top \mathrm{f}($**'It was red',**  $)$

$\mathrm{w}^\top \mathrm{f}($**'It was red',**  $) \; > \; \mathrm{w}^\top \mathrm{f}($**'It was red',**  $)$

…

$\mathrm{w}^\top \mathrm{f}($**'It was red',**  $) \; > \; \mathrm{w}^\top \mathrm{f}($**'It was red',**  $)$

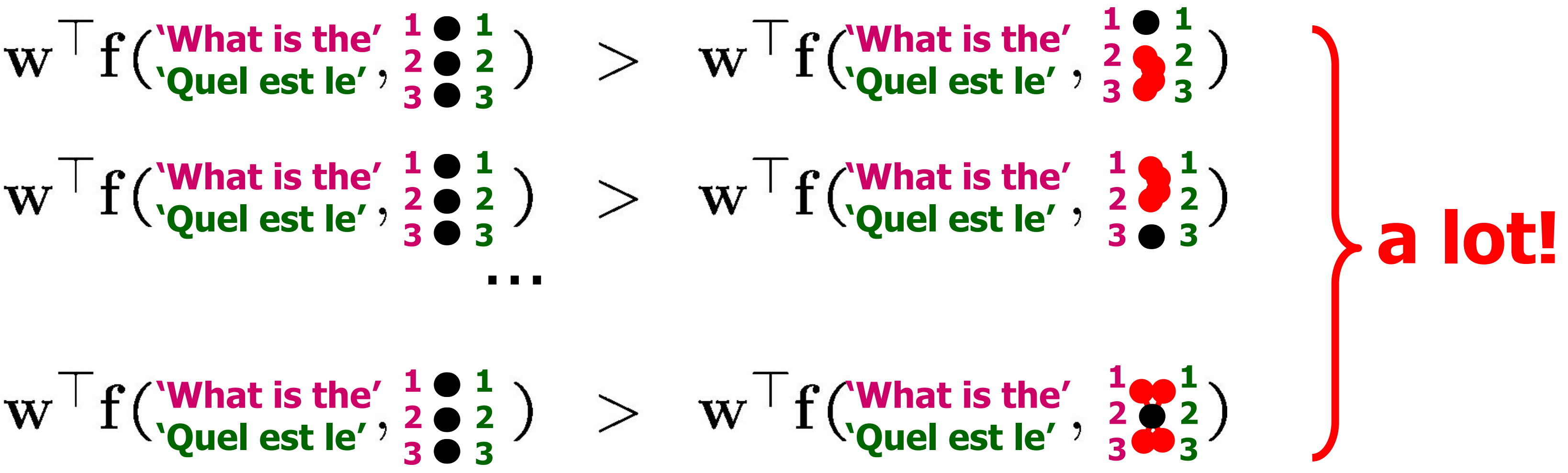
} **a lot!**

Alignment example

- We want:

$$\arg\max_y \; w^\top f\left(\begin{array}{c}\text{'What is the'}\\ \text{'Quel est le'}\end{array}, y\right) = \begin{array}{ccc}1 & \bullet & 1\\ 2 & \bullet & 2\\ 3 & \bullet & 3\end{array}$$

- Equivalently:



Cutting Plane

- A constraint induction method [Joachims et al 09]
  - Exploits that the number of constraints you actually need per instance is typically very small
  - Requires (loss-augmented) primal-decode only

- Repeat:
  - Find the most violated constraint for an instance:

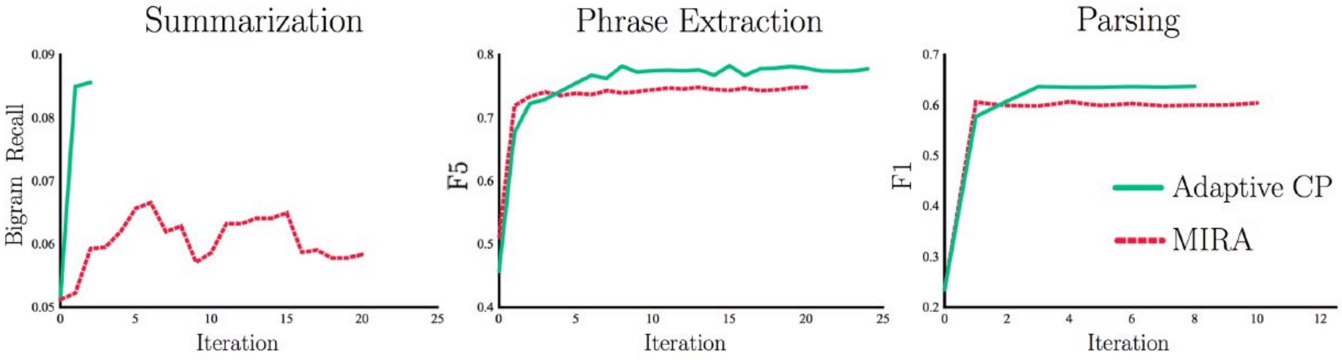
$$\forall \mathbf{y} \quad \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) \geq \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) + \ell_i(\mathbf{y})$$

$$\arg\max_{\mathbf{y}} \mathbf{w}^\top \mathbf{f}_i(\mathbf{y}) + \ell_i(\mathbf{y})$$

  - Add this constraint and resolve the (non-structured) QP (e.g. with SMO or other QP solver)

Cutting Plane (Dual)

- Some issues:
  - Can easily spend too much time solving QPs
  - Doesn't exploit shared constraint structure
  - In practice, works pretty well; fast like perceptron/MIRA, more stable, no averaging



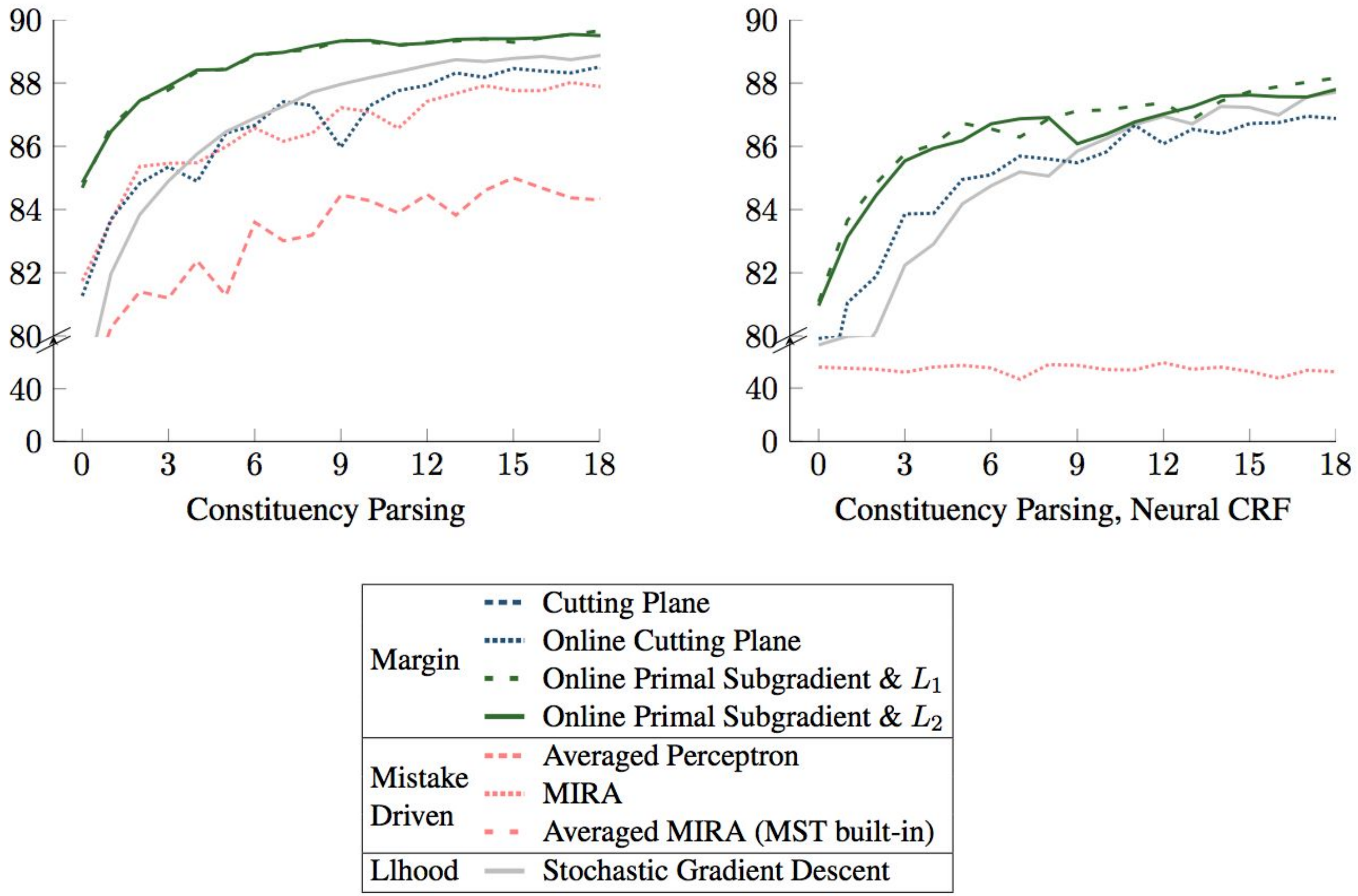
Likelihood, Structured

$$L(\mathbf{w}) = -k||\mathbf{w}||^2 + \sum_i \left(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y}_i^*) - \log \sum_{\mathbf{y}} \exp(\mathbf{w}^\top \mathbf{f}_i(\mathbf{y})) \right)$$

$$\frac{\partial L(\mathbf{w})}{\partial \mathbf{w}} = -2k\mathbf{w} + \sum_i \left(\mathbf{f}_i(\mathbf{y}_i^*) - \sum_{\mathbf{y}} P(\mathbf{y}|\mathbf{x}_i)\mathbf{f}_i(\mathbf{y}) \right)$$

- Structure needed to compute:
  - Log-normalizer
  - Expected feature counts
    - E.g. if a feature is an indicator of DT-NN then we need to compute posterior marginals P(DT-NN|sentence) for each position and sum
- Also works with latent variables (more later)

Comparison

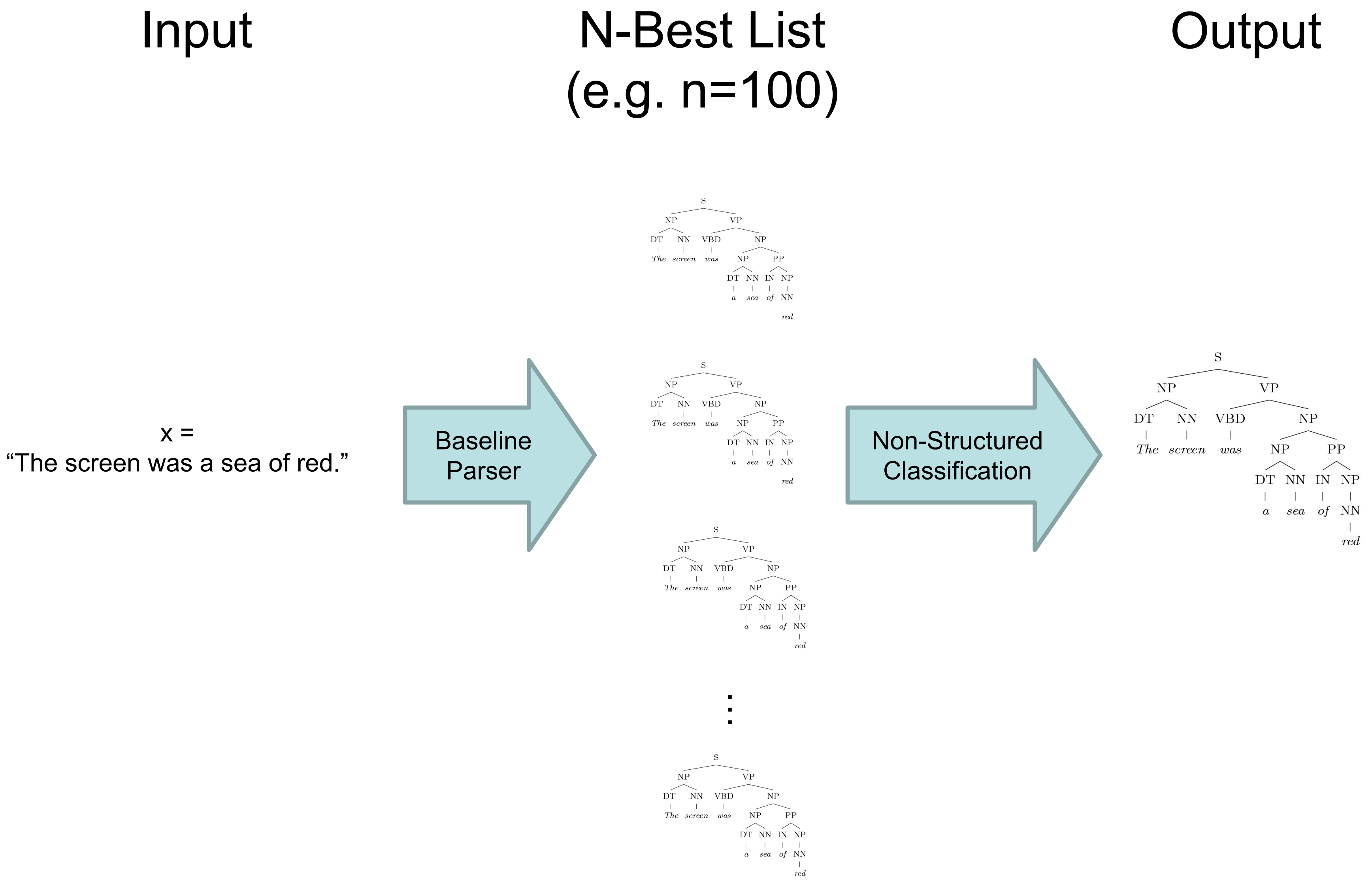


Constituency Parsing

Constituency Parsing, Neural CRF

Option 0: Reranking

[e.g. Charniak and Johnson 05]



Input

N-Best List
(e.g. n=100)

Output

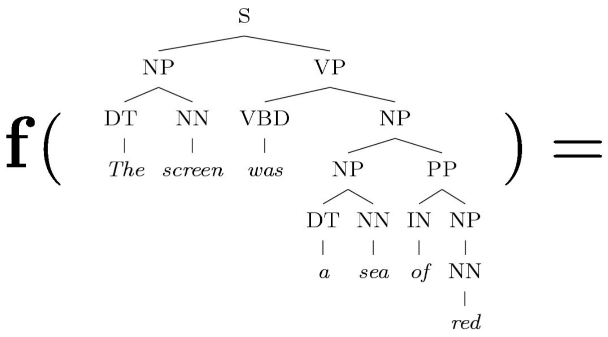
x =
“The screen was a sea of red.”

Baseline Parser

Non-Structured Classification

Reranking

- Advantages:
    - Directly reduce to non-structured case



- Disadvantages:
    - Stuck with errors of baseline parser
    - Baseline system must produce n-best lists
    - But, feedback is possible [McCloskey, Charniak, Johnson 2006]

M3Ns

- Another option: express all constraints in a packed form
  - Maximum margin Markov networks [Taskar et al 03]
  - Integrates solution structure deeply into the problem structure

- Steps
  - Express inference over constraints as an LP
  - Use duality to transform minimax formulation into min-min
  - Constraints factor in the dual along the same structure as the primal; alphas essentially act as a dual “distribution”
  - Various optimization possibilities in the dual

Example: Kernels

- Quadratic kernels

$$K(x, x') = (x \cdot x' + 1)^2$$

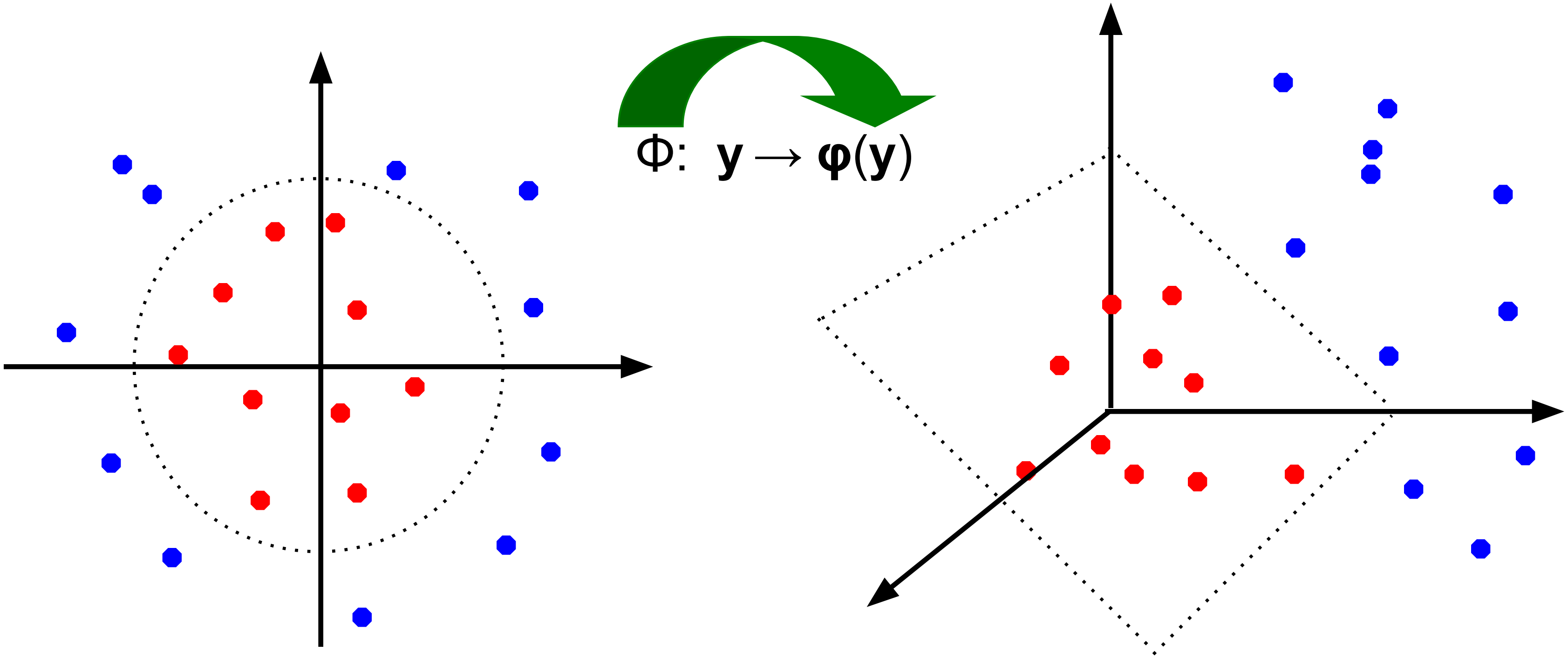
$$= \sum_{i,j} x_i x_j \, x'_i x'_j + 2 \sum_i x_i \, x'_i + 1$$

$$\Downarrow$$

$$K(\mathbf{y}, \mathbf{y}') = (\mathbf{f}(\mathbf{y})^\top \mathbf{f}(\mathbf{y}') + 1)^2$$

Non-Linear Separators

- Another view: kernels map an original feature space to some higher-dimensional feature space where the training set is (more) separable



Why Kernels?

- Can’t you just add these features on your own (e.g. add all pairs of features instead of using the quadratic kernel)?
  - Yes, in principle, just compute them
  - No need to modify any algorithms
  - But, number of features can get large (or infinite)
  - Some kernels not as usefully thought of in their expanded representation, e.g. RBF or data-defined kernels [Henderson and Titov 05]

- Kernels let us compute with these features implicitly
  - Example: implicit dot product in quadratic kernel takes much less space and time per dot product
  - Of course, there’s the cost for using the pure dual algorithms…